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Fixed Points in Intutionistic Fuzzy Metric Spaces for Weakly Compatible Maps

Jitendera Airan , Geeta Modi

Abstract- In this paper, we prove some fixed point theorems for weakly compatible maps in Intutionistic Fuzzy metric space but without assuming the completeness of the space or continuity of mapping involved.

Mathematics subject classification- 47H10, 54A40, 54E99

Keywords- Intutionistic Fuzzy metric space, non compatible maps, weakly compatible maps, Common fixed point , E.A. property.

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1 INTRODUCTION:-

The concept of Fuzzy set was initially introduced by Zadeh[16] as a new way to represent vagueness in everyday life. Subsequently, it was developed extensively by many authors and used in various fields. In 1986 Jungck[6] introduced the notion of compatible maps for a pair of self maps. The study of common fixed point of non compatible mappings is also interesting. Pant [8-10] initiated work along these lines by employing the notion of point wise R-weak commutativity. Atanassov [3] introduced and studied the concept of Intuitionistic Fuzzy sets. Turkoglu et al [14] further formulated the notion of weakly commuting and R-weakly commuting mappings in Intuitionistic fuzzy metric spaces and proved the Intuitionistic fuzzy version of Pant's theorem [8]. Gregori et al. soadati and Park studied the concept of intuitionistic fuzzy metric space and its application.

In the present paper we prove intuitionistic fuzzy version of some common fixed point theorem proved by V.S.Chouhan, V.H.Badsah & M.S.Chauhan [4].

2 DEFINITIONS:-

DEF2.1 A binary operation * : [0,1]x[0,1]→[0,1] is called a continuors t-norm if ([0,1],*) is an abelian topological monoid with the unit 1 such that a*b≤c*d whenever a≤c and b≤d for all a,b,c,d∈ [0,1].

DEF2.2 A binary operation Δ: [0,1]x[0,1]→[0,1] is a continuous t-conorm if Δ satisfied the following conditions :

1. Δ is commutative and associative.
2. Δ is continuous.
3. aΔ0=a ∀a∈ [0,1].
4. aΔb ≤ cΔd whenever a≤c and b≤d for all a,b,c,d∈ [0,1].

DEF2.3 A 5-tuple (X,M,N,*,Δ) is said to be an intuitionistic fuzzy metric space (shortly I.F.M.S.) if X is an arbitrary set, * is a continuous t-norm, Δ is a continuous t-conorm and M,N are fuzzy sets on X^2x(0,∞) satisfying the following conditions :

For all x, y, z ∈X and s, t>0 ;

1. M(x,y,t)+N(x,y,t)≤1
2. M(x,y,0) = 0
3. N(x,y,0) = 1
4. M(x,y,t) = 1 iff x = y
5. N(x,y,t) = 0 iff x = y
6. M(x,y,t) = M(y,x,t)
7. N(x,y,t) = N(y,x,t)
8. M(x,y,t)*M(y,z,s)≤M(x,z,t+s)
9. N(x,y,t)*N(y,z,s)≥N(x,z,t+s)
10. M(x, y, .) : [0,∞] →[0, 1] is left continuous.
11. N(x, y, .) : [0,∞] →[0, 1] is right continuous.
12. $\lim_{t\to\infty} M(x,y,t) = 1$
13. $\lim_{t\to\infty} N(x,y,t) = 0$

Then (M,N) is called an intutionistic fuzzy metric on X. M(x,y,t) and N(x,y,t) denote the degree of nearness and non nearness b/w x and y w.r.to t respectively.

Remark : Every fuzzy metric space (X,M,*) is an intutionistic fuzzy M.S. of the form (X,M,1-M,*,Δ) such that t-norm * and t-conorm Δ are associated as $x \Delta y = 1-[(1-x)*(1-y)]$ for all $x, y \in X$, but converse is not true.

DEF2.4 Let f & g be two maps from an intuitionistic fuzzy metric space $(X,M,N,*,\Delta)$ into itself. Then f & g are said to be compatible if for all t>0

$$\lim_{n\to\infty} M(fgx_n, gfx_n, t) = 1 \quad \&$$

$$\lim_{n\to\infty} N(fgx_n, gfx_n, t) = 0$$

Whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$ for some $z \in X$.

DEF2.5 : Two self maps f & g are said to be weakly compatible if they commute at all coincidence points.

The concept of weak compatibility is most general among all the commutativity concepts. Clearly each pair of compatible self maps is weakly compatible but the converse is not always true.

DEF 2.6 Let S and T be two self maps of a fuzzy metric space(S,M,*) we say that S and T satisfy E.A .Property, if there exists a sequence $\{x_n\}$ in X such that $Sx_n, Tx_n \to x_o$ as $n\to\infty$, for some $x_o \in X$, i.e. $(Sx_n, x_o, t) = (Tx_n, x_o, t) \to 1$ as $n\to\infty$ for some $t \in X$.

3. MAIN RESULTS

THEOREM 3.1 Let f & g be two weak compatible self maps of a intutionistic fuzzy metric space $(X,M,N,*,\Delta)$ satistying the property (E.A.) and

(i) $fx \subset gX$,

(ii) $M(fx,fy,kt) \geq M(gx,gy,t)$, $k \geq 0$

(iii) $N(fx,fy,kt) \leq N(gx,gy,t)$, $k \geq 0$

(iv) $M(fx,ffx,t) > \min\{M(gx,gfx,t), M(fx,gx,t), M(f^2x,gfx,t), M(fx,gfx,t), M(gx,f^2x,t)\}$

(v) $N(fx,ffx,t) < \min\{N(gx,gfx,t), N(fx,gx,t), N(f^2x,gfx,t), N(fx,gfx,t), N(gx,f^2x,t)\}$

Whenever $fx \neq f^2x$.

If the range of f or g is a complete subspace of X, then f and g have a common fixed point.

PROOF. Since f and g are satisfy the property (E.A), there exists a sequence $\{x_n\}$ in X such that

$fx_n, gx_n \to z$ as $n\to\infty$, for some $z \in X$.

since $z \in fX$ and $fX \subset gX$, there exists some point u in X such that $z=gu$ where $gx_n \to z$ $n\to\infty$,

If $fu \neq gu$ then

$M(fx_n,fu,kt) \geq M(gx_n,gu,t)$ and $N(fx_n,fu,kt) \leq N(gx_n,gu,t)$

Taking limit $n\to\infty$, we get

$M(gu,fu,kt) \geq M(gu,gu,t)$ and $N(gu,fu,kt) \leq N(gu,gu,t)$

Which implies $M(gu,fu,kt) \geq 0$ and $N(gu,fu,kt) \leq 0$

Hence $fu=gu$

Since f and g are weakly compatible. So $fgu=gfu$

And therefore $fgu=gfu=ggu$.

If $ffu \neq fu$ then by inequality (iv) and (v)

$M(fu,ffu,t) > \min\{M(gu,gfu,t), M(fu,gu,t), M(f^2u,gfu,t), M(fu,gfu,t), M(gu,f^2u,t)\}$

$=\min\{M(fu,ffu,t), M(fu,fu,t), M(f^2u,ffu,t), M(fu,ffu,t), M(fu,f^2u,t)\}$

$=\min\{M(fu,ffu,t)\}$

$=M(fu,ffu,t)$

Also

$N(fu,ffu,t) < \min\{N(gu,gfu,t), N(fu,gu,t), N(f^2u,gfu,t), N(fu,gfu,t), N(gu,f^2u,t)\}$

$=\min\{N(fu,ffu,t), N(fu,fu,t), N(f^2u,ffu,t), N(fu,ffu,t), N(fu,f^2u,t)\}$

$=\min\{N(fu,ffu,t)\}$

$=N(fu,ffu,t)$

Which is a contradiction and so $fu=ffu$

And $fu=ffu=fgu=gfu=ggu$.

Hence fu is a common fixed point of f and g.

The case when fX is a complete subspace of X is similar to the above since fX $\subset$ gX,

This completes the proof of the theorem.

To illustrate the theorem we give an example.

EXAMPLE 3.1.1 Let X=[-1,2], d(x,y)=Ix-yI, $\forall$ x,y$\in$,

f,g :X$\rightarrow$X defined by

$$fx = \begin{cases} 1 & \text{if } -1\le x\le 1 \\ \frac{3}{4} & \text{if } 1<x<\frac{5}{4} \\ 1+\frac{x^2}{32} & \text{if } \frac{5}{4}\le x\le 2 \end{cases}$$

$$gx = \begin{cases} 1+\frac{x^2}{4} & \text{if } -1\le x\le 1 \\ 1 & \text{if } x=1 \\ 2 & \text{if } 1<x<\frac{5}{4} \\ 1-\frac{x^2}{8} & \text{if } \frac{5}{4}\le x\le 2 \end{cases}$$

For -1≤x≤1,we have

fx= 1, g(x)= 1+ $\frac{x^2}{4}$

gf(x)=1, fg(x)= $\frac{3}{4}$

then M(fgx,gfx,t) = $\frac{t}{t+\frac{1}{4}}$ and M(fx,gx, $\frac{t}{R}$)= $\frac{t}{t+\frac{x^2R}{4}}$

N(fgx,gfx,t) = $\frac{1/4}{t+\frac{1}{4}}$ and N(fx,gx, $\frac{t}{R}$)= $\frac{\frac{1}{4}x^2R}{t+\frac{x^2R}{4}}$

To test R-weakly commuting, we observe that

M(fgx,gfx,t)≥M(fx,gx, $\frac{t}{R}$) and N(fgx,gfx,t)≤N(fx,gx, $\frac{t}{R}$)

Which gives R≥ $\frac{1}{x^2}$,but there exists no R for x=0 $\in$ [-1,1]

Hence f and g are not R-weakly commuting. However for x=1, we have fx=gx=1 and fgx=gfx=1.

Hence f and g are weakly compatible at x=1, clearly fX $\subset$ gX. Then f and g satisty all the conditions of the above theorem.

Also the above theorem can be proved for k=1.

Theorem 1, has been proved by using the concepts of (E.A.) property which has been introduced in a recent work by Aamri and Moutawakil [1]. They have shown that the (E.A.) property is more general than the notion of noncompatibility. It may, however we observed that by using the notion of noncompatible maps in place of (E.A.) property. In next theorem we will show that if we take noncompatible maps in place of (E.A.) property we can show in addition that the mappings are discontinuous at the common fixed point and thus find out an answer in fuzzy metric space to the problem of Rhoades[11].

THEOREM 3.2 Let f and g be two non compatible weakly compatible self mappings of a intutionistic fuzzy metric space (X,M,N,*,Δ) such that

(i) $fX \subset gX$,

(ii) M(fx,fy,kt)≥M(gx,gy,t), k≥0,

(iii) N(fx,fy,kt)≤N(gx,gy,t), k≥0,

(iv) M(fx, ffx, t)>min{M(gx,gfx,t),M(fx,gx,t),M(f^2x,gfx,t),M(fx,gfx,t),M(gx,f^2x,t)},

(v) N(fx, ffx, t)<min{N(gx,gfx,t),N(fx,gx,t),N(f^2x,gfx,t),N(fx,gfx,t),N(gx,f^2x,t)},

Whenever fx≠f^2x.

If the range of f or g is a complete subspace of X, then f and g have a common fixed point and the fixed point is the point of discontinuity.

PROOF. Since f and g are non compatible maps, there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z \quad \text{...........................(1)}$$

For some z in X, but either $\lim_{n\to\infty}(fgx_n,gfx_n,t)\neq 1$ or the limit does non exists.

Since $z\in fX$ and $fX \subset gX$, there exists some point u in X such that z = gu,

Where $z = \lim_{n\to\infty} gx_n$.

We claim that fu = gu. Suppose that fu≠gu then

$M(fx_n,fu,t)\geq M(gx_n,gu,t)$ and $N(fx_n,fu,t)\leq N(gx_n,gu,t)$

Taking limit $n\to\infty$, we get

M(gu,fu,t)≥M(gu,gu,t) and N(gu,fu,t)≤N(gu,gu,t)

Hence fu = gu

Since f and g are weak compatible so fgu = gfu

Therefore ffu = fgu = ggu

Suppose that ffu ≠ fu then by (iv) and (v)

M(fu,ffu,t)>min{M(gu,gfu,t),M(fu,gu,t),M(f^2u,gfu,t),M(fu,gfu,t),M(gu,f^2u,t)}

=min{M(fu,ffu,t),M(fu,fu,t),M(f^2u,ffu,t),M(fu,ffu,t),M(fu,f^2u,t)}

=min{M(fu,ffu,t)}

= M(fu,ffu,t) and

N(fu,ffu,t)<min{N(gu,gfu,t),N(fu,gu,t),N(f^2u,gfu,t),N(fu,gfu,t), N(gu,f^2u,t)}

=min{N(fu,ffu,t),N(fu,fu,t),N(f^2u,ffu,t),N(fu,ffu,t),N(fu,f^2u,t)}

=min{N(fu,ffu,t)}

= N(fu,ffu,t)

Which is a contradiction and so fu = ffu

And fu = ffu = fgu = gfu = ggu

Hence fu is a common fixed point of f and g.

The case when fX is a complete subspace X is similar to the above since $fX\subset gX$.

We now show that f and g are discontinuous at the common fixed point z = fu = gu. If possible, suppose f is continuous, then considering the sequence $\{x_n\}$ of (1) we get $\lim_{n\to\infty} ffx_n = fz = z$.

Since f and g are weakly compatible so ffu = gfu so fz = gz

$ffx_n = gfx_n$ taking limit $n\to\infty$, we get $fz= \lim_{n\to\infty} gfx_n$ or $z = \lim_{n\to\infty} gfx_n$

This , in turn yields,

$$\lim_{n\to\infty}(fgx_n, gfx_n, t) = 1$$

This contradicts the fact that $\lim_{n\to\infty}(fgx_n, gfx_n, t) \neq 1$ or not exist.

Hence f is discontinuous at the fixed point. Similarly we can prove g is discontinuous at the fixed point.

This completes the proof of the theorem.

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